Vector Time Series and Its Properties

Abstract

Productivity of a region can be treated as a vector time series. In this article, we have treated productivity of kharif jawar of Marathwada of Maharastra state as a vector $\bar{r} = (X_1, X_2, \dots X_5)$. Where X_1 = productivity at Aurangabad, X_2 productivity at Parbhani, X_3 = productivity at Osmanabad, X_4 = productivity at Beed and X_5 = productivity at Nanded. Thus, we get a vector time series, $\bar{T} = (r_{ij})$, i = 1, 2 , …….n years, j = 1,2….5 districts. This opens up very interesting questions. How are the properties of T related to component time series?.

A preliminary discussion of properties of vector time series and possible testing methodology for stationary property precedes the actual application to regional productivity data.

Keywords: Time Series, Vector Time Series, Regression Analysis, Auto Covariance, Auto-Correlation.

B. L. Bable
Head,
Deptt.of Statistics,
Dnyanopasak Mahavidyalaya,
Parbhani, Maharastra

D. D. Pawar
Professor and Chairman,
Deptt. of Statistics,
N.E.S. Science College,
Nanded, Maharastra

Introduction

Vector time series can occur naturally in real life. For example, if we consider the productivity of kharif jawar over a region, where productivity is recorded over a cluster of recording stations, we get a vector productivity time series. To what extent the properties of component time series determine the properties of the regional vector time series is worth looking into.

In what follows are first discussed in relation to, few properties of vector time series and then tried to compute the same for the regional annual productivity of kharif jawar record of Marathwada by using data from 1976 to 2002.

Objectives of the Study

1. To develop theory of Vector time series. Specially improving the theorems which characterize vector time series.
2. To develop algorithms for analyzing vector time series, which use the characterizing theorems.
3. By using data from Marathwada Region for validating the algorithms, and testing the methods.
4. To interpret the results of characterizations, in real economic and social terms.

The main purpose of this work is to summarize the research work carried out on the above given objectives and to draw useful conclusions on the basis of auto regressive time series analysis. A way to check trends and randomness in the data scalar as well as vector time series by using properties of auto covariance.

Basic Concepts

Basic definitions and few properties of vector time series are given in this section.

Definition 2.1: A Random Vector

A random vector, $\bar{X} = (X_1, X_2, \dots X_K)$ is a single valued function whose domain is Ω, whose range is in Euclidean n-space R^n and which is B-measurable, i.e. for every subset R C R^n {ω Є Ω | X_1(ω) … X_K(ω) Є R}Є B. A random vector will also be called an K- dimensional random variable or a vector random variable.

If $X_1, X_2 \dots X_K$ are k random variables and $\bar{X} = (X_1, X_2, \dots X_K)$ is a random vector , [18].

Definition 2.2: A Vector Time Series

Let (Ω, C, P) be a probability space; with Ω sample space; C = σ(Ω). Let T be an index set and N = {1,2... k}. A real valued vector time series is a real valued function $X_{it}(\omega)$, i = 1,2…k defined on N ×T ×Ω such that for each fixed t Є T, i Є N, $X_{it}(\omega)$ is a random variable on (Ω, C, P).

A vector time series can be considered as a collection {X_{it} : t Є T }, i =1,2 ... k of random variables [14].

Definition 2.3: Stationary Vector Time Series

A process whose probability structure does not change with time is called stationary. Broadly speaking a vector time series is said to be stationary, if there is no systematic change in mean i.e. no trend. There is no systematic change in variance.

Let $\bar{X} = (x_1, x_2, \ldots x_n)$ be realizations of random variables $(X_1, X_2, \ldots X_K)$.

Definition 2.4: Strictly Stationary Vector Time Series

A vector time series is called strictly stationary, if their joint distribution function satisfy

$$F_{(\bar{x})}\, X_{1t}\, X_{2t} \ldots X_{kt} = F_{(\bar{x})}\, X_{1t+h}\, X_{2t+h} \ldots X_{kt+h} \quad \ldots (1)$$

Where, the equality must hold for all possible sets of indices it and (it + h) in the index set. Further the joint distribution depends only on the distance h between the elements in the index set and not on their actual values.

Main Results

Theorm 3.1

If $\{X_{it} : t \in T\}$, i =1, 2,…k is strictly vector time series with $E\{X_{it}\} < \alpha$ and

$E\{X_{it} - \mu\} < \beta$ then,

$E\{X_{it}\} = E\{X_{it+h}\}$, for all it, h

and $E[\{X_{it} - \mu_i\}\{X_{jt} - \mu_j\}] = E[\{X_{it+h} - \mu_i\}\{X_{jt+h} - \mu_j\}]$, for all it, h } …(2)

Proof

Proof follows from definition (2.4).

In usual cases above equation (2) is used to determine that a vector time series is stationary.

Definition 3.1: Weakly Stationary Vector Time Series

A vector time series is called weakly stationary if

1. The expected value of X_{it} is a constant for all it.
2. The covariance matrix of $(X_{1t}, X_{2t}, \ldots X_{kt})$ is same as covariance matrix of

$(X_{1t+h}, X_{2t+h}, \ldots X_{kt+h})$.

A look in the covariance matrix $(X_{1t}\ X_{2t} \ldots X_{kt})$ would show that diagonal terms would contain terms covariance (X_{it}, X_{it}) which are essentially variances and off diagonal terms would contains terms like covariance (X_{it}, X_{jt}).Hence, the definitions to follow assume importance . Since these involve elements from the same set $\{X_{it}\}$, the variances and co- variances are called auto-variances and auto-co variances .

Definition 3.2: Auto-Covariance Function

The covariance between $\{X_{it}\}$and $\{X_{it+h}\}$ separated by h time unit is called auto-covariance at lag h and is denoted by $\Gamma_{ij}(h)$.

$$\Gamma_{ij}(h) = \mathrm{cov}(X_{it}, X_{jt+h}) = E\{X_{it} - \mu_i\}\{X_{jt+h} - \mu_j\} \quad \ldots (3)$$

The matrix $\Gamma\,\bar{h} = \Gamma_{ij}(h)$ is called the auto covariance matrix function.

Definition 3.3: The Auto Correlation Function

The correlation between observation which are separated by h time unit is called auto-correlation at lag h. It is given by

$$P_{ij}(h) = \frac{E\{X_{it} - \mu_i\}\{X_{jt+h} - \mu_j\}}{[E\{X_{it} - \mu_i\}^2 E\{X_{jt+h} - \mu_j\}^2]^{1/2}} \quad \ldots (4)$$

$$= \frac{\Gamma_{ij}(h)}{[\Gamma_{ii}(h)\ \ \Gamma_{jj}(h)]^{1/2}}$$

Where μ_i is the mean of i^{th} component time series.

Remark 3.1

For a vector stationary time series the variance at time (it+h) is same as that at time it. Thus, the auto correlation at lag h is

$$P_{ij}(h) = \frac{\Gamma_{ij}(h)}{\Gamma_{ii}(0)} \quad \ldots (5)$$

Remark 3.2

For h = 0, we get $\rho_{ij}(0) = 1$.

For application attempts have been made to establish that productivity at certain districts of Marathwada satisfy equation (1) and (5).

Definition 3.4: Positive Semi-Definite

A function f(x) defined for $x \in X$ is said to be positive semi-definite if it satisfies

$$\sum_{J=i}^{n} \sum_{k=1}^{n} a_k^T f(t_j - t_k)\, a_j \geq 0,$$

For any set of real vectors $(a_1, a_2, \ldots a_n)$ and any set of indices $(t_1, t_2 \ldots t_n) \in T$ such that $(t_j - t_k) \in X$.

Theorem 3.2

The covariance function of vector stationary time series $\{X_{it} : t \in T\}$ is positive semi-definite function in that

$$\sum_{J=1}^{n} \sum_{k=1}^{n} a_k^T \Gamma(t_j - t_k)\, a_j \geq 0,$$

For any set of real vectors $(a_1, a_2, \ldots a_n)$ and any set of indices $(t_1, t_2 \ldots t_n) \in T$.

Proof

The result can be obtained by evaluating the variance of

$$X = \sum_{j=1}^{n} a_j^T X_{tj}.$$

For this without loss of generality $E(X_{tj}) = 0$. It shows that the variance of a random variable is non-negative i.e. $V(X) \geq 0$.

$$V(X) = V(\Sigma\ a_j^T X_{tj}) \geq 0$$

$$= E\left(\sum_{j=1}^{n} a_j^T X_{tj}\right)\left(\sum_{k=1}^{n} a_j^T X_{tj}\right)^T \geq 0,$$

$$= \sum_{J=1}^{n} \sum_{k=1}^{n} a_j^T a_k\ E\{X_{tj} X_{tk}\} \geq 0,$$

$$= \sum_{J=1}^{n} \sum_{k=1}^{n} a_k^T \Gamma(t_j - t_k)\ a_j \geq 0 \quad \ldots (6)$$

Hence proved.

Theorem 3.3

$|\rho_{12}(h)| \le 1$.

Proof

If we set n = 2, in the equation (6) to obtain,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_j^T \Gamma(t_i - t_j)\, a_i = a_1^2 \Gamma_{11}(0) + a_2^2 \Gamma_{22}(0) + 2a_1 a_2^T \Gamma(t_1-t_2) \ge 0.$$

$$a_1^2 \Gamma_{11}(0) + a_2^2 \Gamma_{22}(0) \ge -2a_1 a_2^T \Gamma_{12}(t_1-t_2),$$

since $\Gamma_{11}(0) = \Gamma_{22}(0)$

$$1/2(a_1^2 + a_2^2) \ge \frac{-a_1 a_2^T \Gamma_{12}(t_1-t_2)}{\Gamma_{11}(0)}$$

Now, let $a_1 = a_2 = 1$ and $t_1 - t_2 = h$,

$$1 \ge \frac{-\Gamma 12(h)}{\Gamma_{11}(0)} = -\rho_{12}(h) \qquad \text{..... (7)}$$

Similarly, $-a_1 = a_2 = 1$; it shows that

$$P_{12}(h) \le 1 \qquad \text{... (8)}$$

From (7) and (8) we get

$|\rho_{12}(h)| \le 1$.

Hence proved.

Theorem 3.4

The auto covariance matrix of vector stationary time series is an even function of h. i.e., $\Gamma_{ij}(h) = \Gamma_{ij}(-h)^T$.

Proof

Here,

$Cov(X_i, Y_{i+1}) = \{\Sigma X_i Y_{i+1} - 1/n \Sigma X_i \Sigma Y_i\}/n$,

If X_i, Y_i are different series.

$Cov(X_i, Y_{i+1}) \ne Cov(X_i, Y_{i-1})$

i.e. $\{\Sigma X_i Y_{i+1} - 1/n\Sigma X_i \Sigma Y_i\}/n \ne \{\Sigma X_i Y_{i-1} - 1/n \Sigma X_i \Sigma Y_{i+1}\}/n$

$\therefore X_1 Y_2 \ne X_2 Y_1$

When X_i, Y_i are identical series

$\Gamma(1) = \Gamma(-1)$,

Otherewise not true.

Hence, $\Gamma_{ij}(h) = \Gamma_{ij}(-h)^T$ proved.

Theorem 3.5

Let X_{it}'sbe independently and identically distributed with $E(X_{it}) = \mu_I$ and $var(X_{it}) = \sigma_i^2$ then

$$\Gamma_{ij}(t,k) = E(X_{it}, X_{jk}) = \sigma_i^2, \quad t = k$$
$$= 0, \quad t \ne k$$

This process is stationary in the strict sense.

Testing Procedure

Using the model for table-5.1B

$X_i = (\beta_0)_i + (\beta_1)_i t_i + \in_i, \quad i = 1, 2, \dots 5$ . . . (9)

Where

1. X_i are annual productivity series $X_i(t)$, i = 1, 2 … 5 for five districts.
2. t_i are the time (in years) variable.
3. $\in_i$, are a random error term normally distributed as mean zero and variance σ^2, i.e $\in_i \sim N(0, \sigma^2)$.

Productivity X_i (mm) are the dependent variables and time t_i (in years) are independent variables.

Using the model for table-5.2C and 5.3C

$\Upsilon_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij}\, h + \in_{ij}$, i =1, 2, … 5 ; j = 1, 2, … 5 … (10)

Where

1. $\Upsilon_{ij}(h)$ are auto-covariance values for individual series and auto-covariance matrices for vector time series .
2. h are the lag values of variable.
3. $\in_i$ are a random error term normally distributed as mean zero and variance σ^2, i.e. $\in_i \sim N(0, \sigma^2)$. $\Upsilon_{ij}(h)$ are the dependent variables and h are independent variables.

$\Upsilon_{ij}(h)$ are the dependent variables and h are independent variables.

Defining $\Upsilon_{ij}(h) = cov(X_i, X_{j+h})$, (i = 1, 2…5; j =1, 2…5) were computed for various values of h by using MS-Excel. Since the total series constituted of 31 values at least 10 values were included in the computation. The relation between $\Upsilon_{ij}(h)$ and h were examined the model in (table-5.2C).

Defining the $\Gamma_{ij}(h)) = cov(X_i, X_{j+h})$, covariance matrix with a stationary time series for observations $\bar{X} = (x_1, x_2, \dots x_n)$ realizations and $\rho_{ij}(h)$ = correlation (X_i, X_{j+h}) correlation matrix with a stationary time series for observations $\bar{X} = (x_1, x_2, \dots x_n)$ realizations , 21 such matrices corresponding to h = 0 to 20 , define one series of matrices each 5×5 , and hence 25 component series were computed . The relation between $\Gamma_{ij}(h)$ and h were examined in the model.

The method of testing intercept $(\beta_0)_{ij} = 0$ and regression coefficient $(\beta_1)_{ij} = 0$, [10]. Null hypothesis for test Statistic used to test and set up.

Inference concerning slope $(\beta_1)_{ij}$

For testing $H_0 : (\beta_1)_{ij} = 0$ Vs $H_1: (\beta_1)_{ij} > 0$ for $\alpha = 0.05$ percent level using t distribution with degrees of freedom is equal to n – 2 were considered.

$$t_{n-2} = \beta_1 / s_{\beta 1}$$

Where β_1 is the slope of the regression line and $s_{\beta 1} = s_e / s_t$ and $s_e = [SSE / n - 2]^{1/2}$,

sum of squares due to errors (SSE) = $(s_t^2 - s_{tx}^2 / s_t^2)$, $s_{tx} = \Sigma(t_i - \bar{t})(X_i - \bar{X})$

$s_t^2 = \Sigma(t_i - \bar{t})^2$; $s_x^2 = \Sigma(X_i - \bar{X})^2$. The residual sum of squares or the sum of squares due to error is SSE .

The relation between $\Upsilon_{ij}(h)$ and h were examined the model in (table-5.2C).

From model, $\Gamma_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij}\, h + \in_i$,

Table-5.3B was obtained by regressing values of $\Gamma_{ij}(h)$ against h, by using this , testing shows that , both the hypothesis $(\beta_0)_{ij} = 0$ and $(\beta_1)_{ij} = 0$ test is not positive. (Table-A1, APPENDIX-A) formed the input for table-5.3B.

Example of Vector Time Series

Regional productivity of kharif jawar data.

Here is a real instance of a vector time series productivity of kharif jawar data of Marathawada region was obtained from five districts, namely Aurangabad, Parbhani, Beed, Osmanabad, and Nanded. The data were collected from Season and Crop Report, Epitome of Agriculture (part-ii) for

Maharastra state and Maharastra Quarterly Bulletin of Economics and Statistics, Bombay [2, 3, 4].

Hence.we have five dimensional time series t_i , i = 1, 2 , 3 , 4 , 5 corresponding to the districts Aurangabad, Parbhani, Beed, Osmanabad and Nanded respectively. Table 5.1, shows the results of descriptive statistics and Table 5.1B, shows linear trend analysis. All the linear trends were found to be not significant.

Over the years many scientists have analyzed rainfall , temperature , humidity ,agricultural area , production and productivity of Marathwada region of Maharastra state, [1 , 5 , 6 , 7, 8 , 9, 11 , 12 , 13 , 15 , 16] . Most of them have treated the time series for each of the revenue districts as independent time series and tried to examine the stability or non-stability depending upon series. Most of the times non- stability has been concluded , and hence possibly any sort of different treatment was possibility never thought of. In this investigation we treat the series first and individual series and then as a vector time series shows that the vector time series are not stable.

Conclusion

For Scalar Time Series

It was observed that t values are therefore significant for the 4 districts, except Beed district i.e. conclude that X_i depend on t for 4 districts [14]. Similarly $\Upsilon_{ij}(h)$ does not depend on h for 4 districts, except Nanded district. The testing shows that, for the hypothesis $(\beta_1)_i = 0$, test is positive for t, for all four districts and for h test is positive for one district.

Generally it is expected that, productivity of kharif jawar over a long period at any region to be not stationary time series. These results conform with the series in Nanded district.

For Vector Time Series

To conclude that a vector time series is not stationary, it is necessary to test association between $\Gamma_{ij}(h)$ and h.

In terms of productivity series, the 6 series have been attained significant. Due to the significant values in productivity the Nanded individually Parbhani/Aurangabad, Parbhani/Osmanabad, Parbhani/Beed, Parbhani/Nanded, Nanded/ Osmanabad in combinations have turned that the variations which are responsible for the non-stationary nature of the series (table 5.3B).

Hence it is concluded that the whole of the regional productivity of kharif jawar are not stationary status and they have set a trend.

Analysis of Regional Productivity

Productivity Time Series for Each District as Scalar Time Series

To begin with, a straight forward analysis of the five productivity series was carried out to test their trends as a scalar time series.

Table-5.1A: Elementary Analysis of District-Wise Productivity per Hector Data (in Tones) of Marathwada Region for 27 Years (1976-2002).

Cities	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Mean	1079.7	986.5	1015.6	874.3	1060.8
S.D.	322.5	398.9	395.4	333.3	439.5
C.V.	29.9	40.4	38.9	38.1	41.4

Table-5.1B:Test of significance for $\beta_1 = 0$ the model : $X_i(t) = (\beta_0)_i + (\beta_1)_i t + \epsilon_i$, i =1, 2,…5

District	Coefficient	Estimate	Standard Error	t Stat	Significance
Aurangabad	β_0	855.7	122.3	6.994	**S**
	β_1	16.0	7.6	**2.095***	NS
Parbhani	β_0	643.1	144.1	4.464	**S**
	β_1	24.5	9.0	**2.728***	**S**
Osmanabad	β_0	706.9	146.5	4.825	**S**
	β_1	22.1	9.1	**2.412***	**S**
Beed	β_0	654.2	127.5	5.129	**S**
	β_1	15.7	8.0	1.975	NS
Nanded	β_0	696.7	160.4	4.342	**S**
	β_1	26.0	10.0	**2.597***	**S**

*t =2.06 is the critical value for 25 d f at 5% L. S. * shows the significant value*

A look at the table (5.1A) shows that all of them have similar values of CV. Which indicates that their dispersion is almost identical. Trends were found to be significant in all four districts except Beed District. A simple look at the mean values shows that a classification as

C1 = { Aurangabad, Osmanabad, Nanded }

C2 = { Parbhani, Beed }, Could be quite feasible.

Trend

Absence of linear trend, which reasonably low cv values can be taken as evidence of series being stationary series individually in Beed district.

Further search for evidences of stability included determination of auto covariance and their dependency on lag variable h (Table 5.2A). As we know a real test for stationary property of the time series can come establishing that the auto covariance do not depend on the lag variable [14].

Using the model for,

$$X_t = C + \phi X_{t-h} + \epsilon_t, \quad h = 0, 1, 2, \dots 20 \qquad \dots (11)$$

Table-5.2A: Auto co-variances: Districtwise, Ordinary Time Series, Lag Values (h = 0 , 1 , 2 ,………20) About Productivity Data

lag h	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
0	103975.2	159151.7	156327.0	111105.2	193135.0
1	10042.7	17567.4	14882.1	6860.1	-13884.5
2	-8031.8	43000.5	21124.9	-12327.9	46248.5
3	26637.8	25092.2	61914.9	34111.1	66202.9
4	7027.9	-52247.5	-26517.7	-8819.5	-19657.5
5	-19239.0	40781.5	29797.7	17320.5	82390.8
6	14742.0	-23250.1	14078.5	3680.4	-48028.8
7	-8155.9	-7740.7	-47851.0	-4383.1	36816.0
8	-37595.9	56787.3	14292.5	686.6	20952.2
9	37768.1	-17516.1	-22715.2	-25868.5	-42296.6
10	21860.5	55604.8	-18361.9	17475.0	56750.6
11	-15115.4	44699.8	66608.0	-9055.6	-6069.2
12	9544.6	12149.2	-24486.2	4329.9	45781.9
13	24073.0	68941.0	26181.2	27420.1	10682.6
14	-20176.0	-23003.0	33319.7	-35027.8	4526.1
15	57675.6	-25.0	20981.5	44178.4	53078.4
16	35802.6	20601.7	45725.5	41736.5	-771.7
17	-31947.6	-83694.9	-243.8	-16181.4	-15197.6
18	-16509.3	15129.9	257.7	29196.8	-47242.8
19	31744.4	-39109.5	-12277.6	-12335.0	3448.8
20	-26161.5	47543.1	-52534.0	-66874.4	-5487.9

Table-5.2B: Correlation coeficient between h and Auto covariance is :

Corr. Coeficient	-0.246	-0.342	-0.393	-0.354	**-0.438***

*Correlation coefficient r = 0.433 is the critical value for 19 d f at 5% L. S. * shows the significant value.*

Table-5.2C:Test of significance for $\beta_1 = 0$ the model : $\Upsilon_i(h) = (\beta_0)_i + (\beta_1)_i h + \varepsilon_i$, (i = 1 , 2 ,…5: h = 0, 1, ……..20)

District	Coeficient	Estimate	Standard Error	t Stat	Significance
Aurangabad	β_0	22781.8	14128.1	1.613	NS
	β_1	-1335.5	1208.5	-1.105	NS
Parbhani	β_0	45393.7	20793.8	2.183	**S**
	β_1	-2822.9	1778.7	-1.587	NS
Osmanabad	β_0	43334.3	18226.6	2.378	**S**
	β_1	-2902.5	1559.1	-1.862	NS
Beed	β_0	27401.8	14456.0	1.896	NS
	β_1	-2039.1	1236.6	-1.649	NS
Nanded	β_0	58552.6	21182.0	2.764	NS
	β_1	-3848.7	1811.9	**-2.124***	NS

*t =2.93 is the crical value for 19 d f at 5% L. S. * shows the significant value*

Correlation's between γ_{ij} (h) and h were found to be significant in Nanded district only, showing that the time series can be reasonably assumed to be not stationary in Nanded district. The coefficient β_1 is significant in Nanded district with negative value showing that it has been experiencing significantly declining productivity over the past years.

Productivity time series of five districts treated as a single vector time series

Out of the 25 components in productivity series 6 series showed significant (coefficients) intercepts and slope. Due to the significant values in productivity the Nanded-0.438* individually Parbhani/ Aurngabad-0.460*, Parbhani/Osmanabad-0.502*, Parbhani/Beed-0.437*, Parbhani/Nanded-0.515*, Nanded/Osmanabad-0.484* in combinations seem to be causing the variations responsible for the non-stationary nature of the series. Hence we may conclude that the regional productivity of kharif jawar is not stationary and must have a trend.

Table=5.3A: Cov.(h, Γ_{ij}(h)) Matrix values about productivity data.

District	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Aurangabad	-48968.4	-78792.2	-83784.8	-70735.4	-81392.5
Parbhani	-51937.9	-103505.6	-103275.0	-88929.3	-103635.9
Osmanabad	-39764.2	-105406.3	-106423.4	-93579.1	-94460.8
Beed	-32422.9	-74668.5	-83668.8	-74766.7	-69376.6
Nanded	-68061.7	-119994.9	-131202.3	-107562.8	-141119.2

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Table-5.3B: $\rho_{ij}(h)$ = Correlation(h, $\Gamma_{ij}(h)$) Matrix Values About Productivity Data

District	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Aurangabad	-0.246	-0.377	-0.397	-0.344	-0.397
Parbhani	**-0.460***	-0.342	**-0.502***	**-0.437***	**-0.515***
Osmanabad	-0.269	-0.365	-0.393	-0.394	-0.410
Beed	-0.212	-0.320	-0.405	-0.354	-0.380
Nanded	-0.405	-0.326	**-0.484***	-0.381	**-0.438***

*Correlation coefficient r = 0.433 is the critical value for 19 d f at 5% L. S. * shows the significant value.*

Table-5.3C: Test of significance for β_1 =0, the model Υ ij(h) = βij (0) + βij (1)h + eij (h) , (i = 1,2…5 ; districts, j = 1,2 …5 ; districts , and h = 0 , 1 2 , ……20 ; Lag values.)

District/District	Coefficient	Estimate	Stand. error	t-stat	Significant (S) or not
Aurangabad/Aurangabad	β_0	22781.8	14128.1	1.613	NS
	β_1	-1335.5	1208.5	-1.105	NS
Aurangabad/Parbhani	β_0	31942.3	14169.7	2.254	S
	β_1	-2148.9	1212.1	-1.773	NS
Aurangabad/Osmanabad		34928.2	14183.7	2.463	S
	β_0	-2285.0	1213.3	-1.883	NS
Aurangabad/Beed	β_1	26684.1	14114.5	1.891	NS
	β_0	-1929.1	1207.4	-1.598	NS
Aurangabad/Nanded	β_1	34788.1	13781.0	2.524	S
	β_0	-2219.8	1178.8	-1.883	NS
Parbhani/Aurangabad	β_0	22709.5	7338.3	3.095	S
	β_1	-1416.5	627.7	-2.257*	S
Parbhani/Parbhani	β_0	45393.7	20793.8	2.183	S
	β_1	-2822.9	1778.7	-1.587	NS
Parbhani/Osmanabad	β_0	43133.6	13000.2	3.318	S
	β_1	-2816.6	1112.0	-2.533*	S
Parbhani/Beed	β_0	34001.5	13385.8	2.540	S
	β_1	-2425.3	1145.0	-2.118*	S
Parbhani/Nanded	β_0	43028.4	12603.8	3.414	S
	β_1	-2826.4	1078.1	-2.622*	S
Osmanabad/Aurangabad	β_0	20949.7	10429.4	2.009	NS
	β_1	-1084.5	892.1	-1.216	NS
Osmanabad/Parbhani	β_0	39119.5	19675.2	1.988	NS
	β_1	-2874.7	1683.0	-1.708	NS
Osmanabad/Osmanabad	β_0	43334.3	18226.6	2.378	S
	β_1	-2902.5	1559.1	-1.862	NS
Osmanabad/Beed	β_0	33508.5	15956.4	2.100	S
	β_1	-2552.2	1364.9	-1.870	NS
Osmanabad/Nanded	β_0	39376.3	15356.7	2.564	S
	β_1	-2576.2	1313.6	-1.961	NS
Beed/Aurangabad	β_0	16551.0	10926.9	1.515	NS
	β_1	-884.3	934.7	-0.946	NS
Beed/Parbhani	β_0	27822.4	16158.3	1.722	NS
	β_1	-2036.4	1382.2	-1.473	NS
Beed/Osmanabad	β_0	33362.1	13831.0	2.412	S
	β_1	-2281.9	1183.1	-1.929	NS
Beed/Beed	β_0	27401.8	14456.0	1.896	NS
	β_1	-2039.1	1236.6	-1.649	NS
Beed/Nanded	β_0	28931.6	12371.0	2.339	S
	β_1	-1892.1	1058.2	-1.788	NS
Nanded/Aurangabad	β_0	31255.3	11232.3	2.783	S
	β_1	-1856.2	960.8	-1.932	NS

Nanded/Parbhani	β_0	50298.3	25413.5	1.979	NS
	β_1	-3272.6	2173.9	-1.505	NS
Nanded/Osmanabad	β_0	55036.1	17356.3	3.171	S
	β_1	-3578.2	1484.7	-2.410*	S
Nanded/Beed	β_0	41816.9	19075.1	2.192	S
	β_1	-2933.5	1631.7	-1.798	NS
Nanded/Nanded	β_0	58552.6	21182.0	2.764	NS
	β_1	-3848.7	1811.9	-2.124*	NS

*t =2.093 is the crical value for 19 d f at 5% L. S. * shows the significant (S) value*

Thus we have a situation where when treated as individual series the Nanded district is not stationary and when treated as vector time series the whole of the vector time series is not stationary. This conforms one of the properties of vector time series,

If one of the component series is non stationary, the vector series as a whole is also a non stationary, [14].

When we think of the productivity of kharif jawar of whole of the region, the district Nanded individually Parbhani/Aurangabad, Parbhani/ Osmanabad, Parbhani/Beed, Parbhani/ Nanded, Nanded/Osmanabad in combinations seem to be causing the variations responsible for the non-stationary nature of the series. Hence we may conclude that the regional productivity of kharif jawar is not stationary and must have a trend.

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